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ABSTRACT 23794

A numerical procedure enabling the analysis of temperature measurements by optical pyrometry is described. The determination of the effective wavelength normally carried out with present methods, and the complicated iterative procedure it entails, are eliminated. A high degree of accuracy is attained with a limited amount of calculation: for three typical commercial color filters, the maximum errors arising from the calculations are between 0.01 and 0.04 degree for a temperature range of 1200-3000° K. /640*

AUTHOR

$x \ll 2\pi(\Delta\lambda L)^{-1/2}$

Optical-pyrometrical temperature measurements are customarily evaluated by a procedure described in Henning (Ref. 1), which requires a knowledge of the effective wavelength of the color filter utilized. This wavelength, however, depends on two temperatures -- the observational temperature and the temperature to be measured. An expression first given by Foote (Ref. 2) for the effective limiting wavelength makes it

*Note: Numbers in the margin indicate pagination in the original foreign text.

possible for the effective wavelength, and thus the unknown temperature also, to be approximated by an iterative procedure. This method offers the inexperienced calculator little more than difficulty; moreover, a large number of numerical integrations must be performed in order to tabulate the effective limiting wavelength.

Objective temperature-measuring devices have recently been in the process of development to an increasing extent, and have already afforded relative accuracies within hundredths of a degree. This is a precision which cannot be guaranteed by the conventional computational method, and is also the reason for the obvious desire for a better method of evaluation. /641

The following describes a new procedure which complies with the increased demands for accuracy, and even reduces the number of integrations which are necessary. Iterations are furthermore eliminated, since the unknown temperature may be specifically calculated. The orderly formula apparatus should also be more easily managed by less practiced calculators.

General Principles

Optical measurement of temperature deals chiefly with a comparison of two radiant temperatures - in general, by attempting by means of variable attenuation to make the weakened radiations seem identical to an observer, or to be so estimated by an objective receiver. The condition for successfully balancing the radiation may be mathematically formulated as follows:

$$D \int_\lambda \varepsilon(\lambda,T)\, P(\lambda,T)\, V(\lambda)\, D(\lambda)\, d\lambda = D' \int_\lambda \varepsilon'(\lambda,T')\, P(\lambda,T')\, V(\lambda)\, D'(\lambda)\, d\lambda. \qquad (1)$$

Here D, D' and $D(\lambda)$, $D'(\lambda)$ denote the transmittancy of neutral and selective attenuating media, respectively; $\varepsilon(\lambda,T)$ and $\varepsilon'(\lambda,T')$ indicate the degree of emission of the radiant body which generally depends on wavelength λ and temperature T or T'; $P(\lambda,T)$ is Planck's function which, with the constant factor omitted, is:

$$P(\lambda,T) = \lambda^{-5}\left[\exp\frac{c_2}{\lambda T} - 1\right]^{-1}. \tag{2}$$

$V(\lambda)$ is the sensitivity of the human eye to brightness, which in objective observation is replaced by the sensitivity of the receiver employed.

Formula (1) is valid if the radiation is not equilibrated directly, but through a secondary norm (the pyrometer bulb). If the left side of Equation (1) is designated by J, the right by J', and the corresponding expression for the secondary norm by J*, the relations

$$J = J^* \tag{3a}$$

$$\text{and} \quad J' = J^* \tag{3b}$$

are valid, whence Equation (1) again ensues.

In order to determine an unknown temperature T, the temperature T' of the comparison radiator must be known, as well as transmittancy D, D' and $D(\lambda)$, $D'(\lambda)$ and sensitivity $V(\lambda)$. A knowledge of the radiant properties of both radiators, described by the degree of emission $\varepsilon(\lambda,T)$, is also required. A black body is generally used as the comparison radiator; therefore, $\varepsilon'(\lambda,T') = 1$. In order to simplify the calculating /642 procedure, let us assume that the second radiator is also a black body whose temperature T is to be determined. Thus, we also have $\varepsilon(\lambda,T) = 1$. Thus, Equation (1) can be reduced to:

$$D\int_\lambda P(\lambda,T)\,V(\lambda)\,D(\lambda)\,d\lambda = D'\int_\lambda P(\lambda,T')\,V(\lambda)\,D'(\lambda)\,d\lambda. \tag{4}$$

Description of the Computational Procedure

For sufficiently small values of $\lambda \cdot T$, Planck's function may be replaced by Wien's approximation:

$$P(\lambda,T) \approx W(\lambda,T) = \lambda^{-5} \exp\left(-\frac{c_2}{\lambda T}\right). \tag{5}$$

If the spectral region in which $D(\lambda)$ or $D'(\lambda)$ differ from zero is likewise so small that variation in λ need not be taken into account, the integrals in Equation (4) are converted into expressions such as:

$$A e^{-\frac{c_2}{\lambda_0 T}} \tag{6}$$

with fixed λ_0. Taking the logarithm of expression (6), we have:

$$\log \int_\lambda P(\lambda,T)\, V(\lambda)\, D(\lambda)\, d\lambda \approx \log A - \frac{Mc_2}{\lambda_0 T}. \tag{7}$$

This procedure may also be regarded as the first approximation of a Taylor expansion whose generalization is:

$$\log \int_\lambda P(\lambda,T)\, V(\lambda)\, D(\lambda)\, d\lambda = \sum_k \frac{a_k}{T^k}. \tag{8}$$

The case in which the series is truncated at k = 3 is to be treated here as a particular case of Equation (8). Then the result derived by taking the logarithm of Equation (4) is:

$$\begin{aligned} &\log D + a_0 + a_1\left(\frac{1}{T}\right) + a_2\left(\frac{1}{T}\right)^2 + a_3\left(\frac{1}{T}\right)^3 \\ &= \log D' + a_0' + a_1'\left(\frac{1}{T'}\right) + a_2'\left(\frac{1}{T'}\right)^2 + a_3'\left(\frac{1}{T'}\right)^3 \end{aligned} \tag{9}$$

or

$$\frac{1}{T} + \alpha\frac{1}{T^2} + \beta\frac{1}{T^3} = \gamma_0 + \gamma_1\frac{1}{T'} + \gamma_2\left(\frac{1}{T'}\right)^2 + \gamma_3\left(\frac{1}{T'}\right)^3 + \delta\log\frac{D'}{D} \tag{10}$$

with

$$\alpha = \frac{a_2}{a_1},\ \beta = \frac{a_3}{a_1},\ \gamma_0 = \frac{a_0' - a_0}{a_1},\ \gamma_1 = \frac{a_1'}{a_1},\ \gamma_2 = \frac{a_2'}{a_1},\ \gamma_3 = \frac{a_3'}{a_1},\ \delta = \frac{1}{a_1}.$$

To determine the unknown temperature T, the right side of Equation (10) must be known. Here it is set equal to X, so that it follows:

$$\frac{1}{T} + \alpha \frac{1}{T^2} + \beta \frac{1}{T^3} - X = 0. \qquad (11)$$

This equation may be solved by approximation and, with the choice /643 of X as the initial value, it assumes the form:

$$\frac{1}{T} = X\,f(X), \qquad (12)$$

where f(X) is close to 1. Thus, the further transformation:

$$T = \frac{1}{X} g(X) \qquad (13)$$

may also be made, and g(X) can also be expanded in a series so that finally, disregarding terms of higher order, we obtain:

$$T^* = AX^{-1} + B + CX^{+1}. \qquad (14)$$

The symbol T* is used to differentiate the computed from the real value of T.

Calculating the Constants

The constants of Equation (9) may be computed from a system of euations of the fourth order. If we introduce the notation:

$$L_n = \log \int_\lambda P(\lambda, T_n)\, V(\lambda)\, D(\lambda)\, d\lambda = \sum_{k=0}^{3} \frac{a_k}{T_n^k} \qquad (15)$$
$$n = 1 \ldots 4,$$

then, with fixed chosen temperatures T_n, the constants are derived as:

$$a_k = \sum_{n=1}^{4} b_{k,n} L_n. \qquad (16)$$

The temperatures may be so combined that the values of $b_{k,n}$ in Equation (16) become very simple.

The constants of Equation (14) may likewise be computed from a system of equations, if X is expressed by $\frac{1}{T}$ according to Equation (11), and three appropriately-chosen temperatures T_m are introduced. If $T_m^* = T_m$, it is then true that :

$$1+\alpha \frac{1}{T_m}+\beta \frac{1}{T_m^2}=A+B\left(\frac{1}{T_m}+\alpha \frac{1}{T_m^2}+\beta \frac{1}{T_m^3}\right)$$
$$+C\left(\frac{1}{T_m}+\alpha \frac{1}{T_m^2}+\beta \frac{1}{T_m^3}\right)^2. \quad (17)$$

Approximations which represent A, B, C as a function of α and β are given below. Temperatures T_1, T_3 and T_4 from equation system (15) are substituted for the values of T_m, for computational purposes. The same patterns are used to figure the primed values a_k' as for the plain values a_k; the values of L are then replaced by the corresponding primed values.

First combination: /644

$$T_1 = 1200\ ^\circ K \quad T_2 = 1500\ ^\circ K \quad T_3 = 2000\ ^\circ K \quad T_4 = 3000\ ^\circ K$$
$$a_0 = 10\,L_4 - 20\,L_3 + 15\,L_2 - 4\,L_1$$
$$a_1 = (26\,L_1 - 93\,L_2 + 114\,L_3 - 47\,L_4)\,10^3$$
$$a_2 = (4\,L_4 - 11\,L_3 + 10\,L_2 - 3\,L_1)\,18 \cdot 10^6$$
$$a_3 = (L_1 - 3\,L_2 + 3\,L_3 - L_4)\,36 \cdot 10^9$$
$$A = 1 + \frac{5}{36}(2\alpha^3 - 3\alpha\beta)\,10^{-9}$$
$$B = \alpha - \frac{31}{36}(2\alpha^3 - 3\alpha\beta)\,10^{-6}$$
$$C = (\beta - \alpha^2) + \frac{60}{36}(2\alpha^3 - 3\alpha\beta)\,10^{-3}$$

Second combination:

$$T_1 = 1000\ ^\circ K \quad T_2 = 1200\ ^\circ K \quad T_3 = 1500\ ^\circ K \quad T_4 = 2000\ ^\circ K$$
$$a_0 = 20\,L_4 - 45\,L_3 + 36\,L_2 - 10\,L_1$$
$$a_1 = (47\,L_1 - 162\,L_2 + 189\,L_3 - 74\,L_4)\,10^3$$
$$a_2 = (5\,L_4 - 14\,L_3 + 13\,L_2 - 4\,L_1)\,18\,10^6$$
$$a_3 = (L_1 - 3\,L_2 + 3\,L_3 - L_4)\,36\,10^9$$
$$A = 1 + \frac{2}{6}(2\alpha^3 - 3\alpha\beta)\,10^{-9}$$
$$B = \alpha - \frac{9}{6}(2\alpha^3 - 3\alpha\beta)\,10^{-6}$$
$$C = (\beta - \alpha^2) + \frac{13}{6}(2\alpha^3 - 3\alpha\beta)\,10^{-3}$$

Third combination:

$$T_1 = 1500\,°K \quad T_2 = 1800\,°K \quad T_3 = 2250\,°K \quad T_4 = 3000\,°K$$

$$a_0 = 20\,L_4 - 45\,L_3 + 36\,L_2 - 10\,L_1$$

$$a_1 = (47\,L_1 - 162\,L_2 + 189\,L_3 - 74\,L_4)\,\frac{3}{2}\,10^3$$

$$a_2 = (5\,L_4 - 14\,L_3 + 13\,L_2 - 4\,L_1)\,\frac{81}{2}\,10^6$$

$$a_3 = (L_1 - 3\,L_2 + 3\,L_3 - L_4)\,\frac{243}{2}\,10^9$$

$$A = 1 + \frac{8}{81}(2\alpha^3 - 3\alpha\beta)\,10^{-9}$$

$$B = \alpha - \frac{54}{81}(2\alpha^3 - 3\alpha\beta)\,10^{-6}$$

$$C = (\beta - \alpha^2) + \frac{117}{81}(2\alpha^3 - 3\alpha\beta)\,10^{-3}$$

Computational Accuracy /645

The accuracy of the calculations depends on the spectral position and the extension of the filter band and on the temperature range or combination. Figure 1 gives the standardized transmittancy curves of three color filters, as utilized in optical pyrometry.

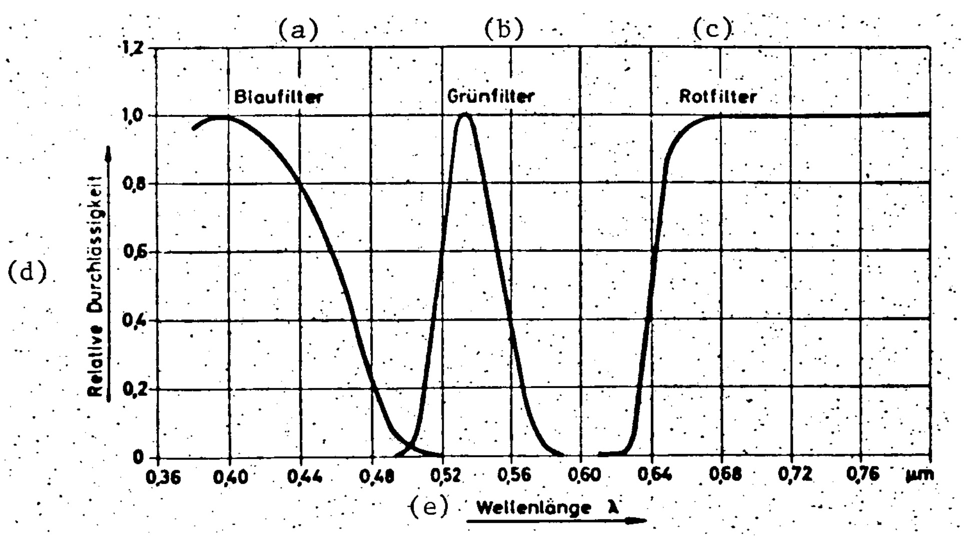

Figure 1

Relative Transmittancy of Three Color Filters
(a)-Blue; (b)-Green; (c)-Red; (d)-Relative Transmittancy; (e)-Wavelength λ

On the assumption that $D(\lambda) = D'(\lambda)$, these values were used to compute the integrals of Equation (4) for various temperatures with fixed $T' = T_{Au} = 1336.15^{\circ}K$. For every T, the pertinent value of D'/D is /646 derived from this, which is substituted in the right side of Equation (10) to calculate X. Because $D(\lambda) = D'(\lambda)$, the following equalities develop:

$$\gamma_0 = 0,\ \gamma_1 = 1,\ \gamma_2 = \alpha,\ \gamma_3 = \beta$$

and thus

$$X = \frac{1}{T_{Au}} + \alpha\frac{1}{T_{Au}^2} + \beta\frac{1}{T_{Au}^3} + \delta\log\frac{D'}{D}.$$

The computed value T* belonging to T then results from Equation /647 (14). Figure 2 gives the differences $\Delta T = T - T^*$ as a function of T or T*, which cannot be differentiated on the axis of the abscissas, for the three color filters and the first combination. With the green and blue filters, ΔT in the 1200-3000°K temperature range remains less than 0.01°; with the red filter, less than 0.04°. Even when the temperature range is in each case exceeded by 200°, all requirements for accuracy are

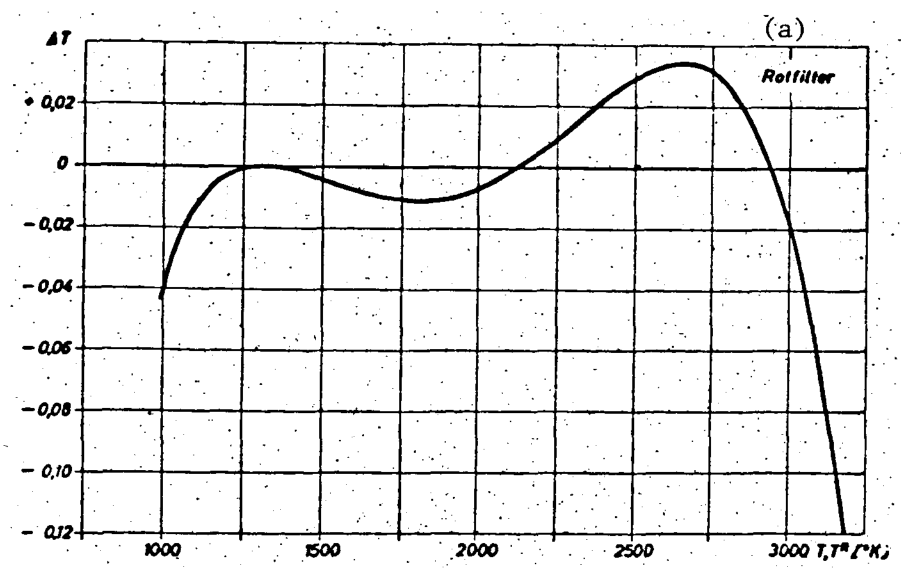

Figure 2a

Figures 2a-c. Computational Errors for Three Color Filters and First Combination

(a)-Red Filter

still fulfilled. Figure 3 gives a similar presentation for the red filter and the second combination. Here ΔT between 1000 and 2000°K remains even less than 0.003°. The third combination, on the contrary, affords no essential improvement in the 2000-3000°K temperature range.

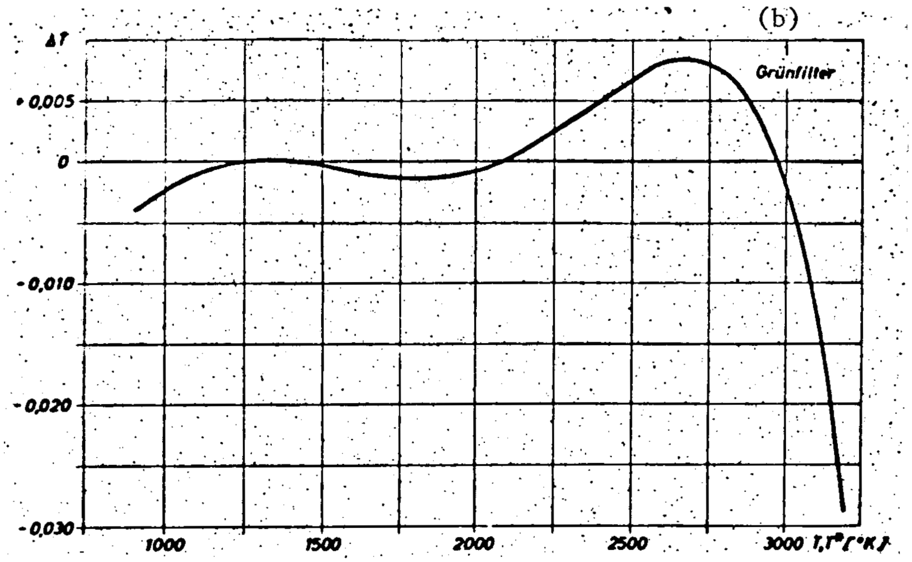

Figure 2b
(b)-Green Filter

Since the temperature of solidifying gold T_{Au} is the basis of the optical temperature scale, the error specification which has just been formulated contains all the necessary information. No error propagation occurs if the values of T* are used for subsequent calculations, as the procedure is applied step by step.

Applications

1. Temperature Determination

(a) Neutral attenuation media.

In this case $D(\lambda) = D'(\lambda)$ and we have:

$$\gamma_0 = 0, \quad \gamma_1 = 1, \quad \gamma_2 = \alpha, \quad \gamma_3 = \beta.$$

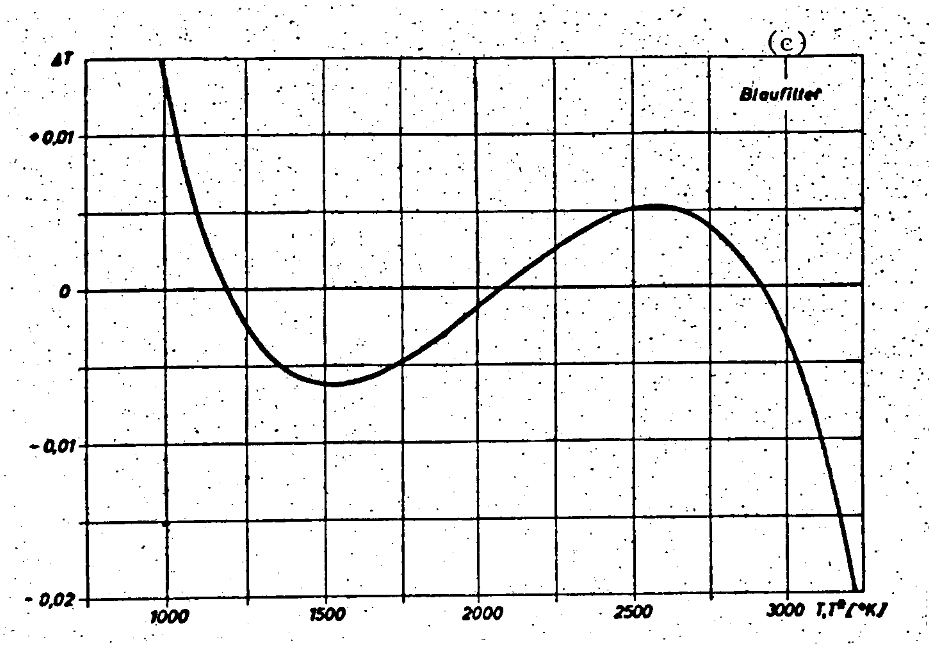

Figure 2c
(c)-Blue Filter

Let a black body have an unknown temperature T, and let its radiation be attenuated with a rotating sector of transmittancy D. /648
Calibration was accomplished without the sector, i.e., D' = 1, and the observation provides observational temperature T'. We then have:

$$X = \frac{1}{T'} + \alpha\left(\frac{1}{T'}\right)^2 + \beta\left(\frac{1}{T'}\right)^3 + \delta \log \frac{1}{D}. \qquad (18)$$

If, on the contrary, radiation of a black body of known temperature T' is attenuated with a sector of transmittancy D', it then corresponds to the unattenuated radiation (D = 1) of a black body of temperature T. Therefore, we have:

$$X = \frac{1}{T'} + \alpha\left(\frac{1}{T'}\right)^2 + \beta\left(\frac{1}{T'}\right)^3 + \delta \log D'. \qquad (19)$$

This case may perhaps occur when a pyrometer is calibrated below the gold point; the computed temperature T* is the one which is to be associated with the measured pyrometer current.

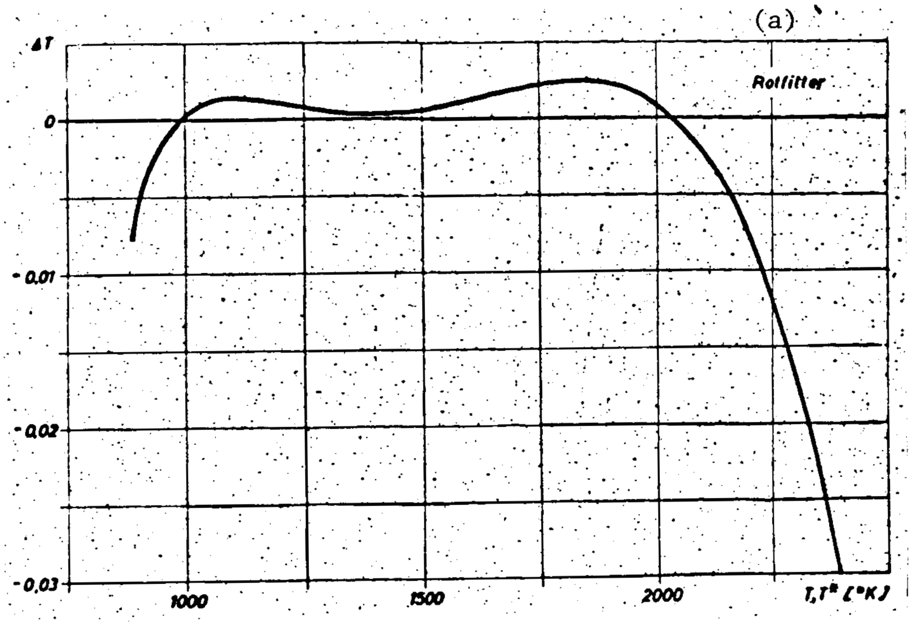

Figure 3

Computational Errors for Red Filter and Second Combination
(a)-Red Filter

(b) Selective attenuation media.

Here $D(\lambda) \neq D'(\lambda)$, but in general $D(\lambda) = D^*(\lambda)D'(\lambda)$. This is, for example, necessary in pyrometers, so that the radiation of the secondary norm under equal conditions (pyrometer current) will remain unchanged and Equations (3a) and (3b) will remain valid.

If the pyrometer is calibrated with a filter of transmittancy $D'(\lambda)$, and in addition a smoked glass of transmittancy $D^*(\lambda)$ is used, then Equation (10) is generally true.

$$X = \gamma_0 + \gamma_1 \frac{1}{T'} + \gamma_2 \left(\frac{1}{T'}\right)^2 + \gamma_3 \left(\frac{1}{T'}\right)^3 + \delta \log \frac{D'}{D}$$

where

$$\gamma_0 = \frac{a_0' - a_0}{a_1}, \gamma_1 = \frac{a_1'}{a_1}, \gamma_2 = \frac{a_2'}{a_1}, \gamma_3 = \frac{a_3'}{a_1}, \delta = \frac{1}{a_1}.$$

If, moreover, no neutral attenuation media are employed, then $D = D' = 1$ and $\log \frac{D'}{D} = 0$. The values of α and β now differ from their values in section (a) - i.e., their values are as computed for the combination of color filter and smoked glass.

2. Transmittancy Measurements

(a) Neutral attenuation media.

In order to determine transmittancy D of a neutral attenuation medium, the temperature T of a black body is measured, and the attenuation medium is then placed in the path of the radiation, and the observational temperature T' derived. We then have:

$$D \int_\lambda P(\lambda, T)\, V(\lambda)\, D(\lambda)\, d\lambda = \int_\lambda P(\lambda, T')\, V(\lambda)\, P(\lambda)\, d\lambda .$$

From Equation (9) it follows that, when $D' = 1$ and $D(\lambda) = D'(\lambda)$ and therefore $a_k = a_k'$:

$$\log D = a_1\left(\frac{1}{T'} - \frac{1}{T}\right) + a_2\left(\frac{1}{T'^2} - \frac{1}{T^2}\right) + a_3\left(\frac{1}{T'^3} - \frac{1}{T^3}\right). \qquad (20)$$

(b) Selective attenuation media. /649

For selective attenuation media of transmittancy $D^*(\lambda)$, their effective transmittancy D_{eff} in the light of a filter of transmittancy $D'(\lambda)$ is defined by the equation

$$\int_\lambda P(\lambda, T)\, V(\lambda)\, D^*(\lambda)\, D'(\lambda)\, d\lambda = D_{eff} \int_\lambda P(\lambda, T)\, V(\lambda)\, D'(\lambda)\, d\lambda .$$

The following is derived from Equation (9), with $D = 1$ and $D' = D_{eff}$:

$$\log D_{eff} = a_0 - a_0' + (a_1 - a_1')\frac{1}{T} + (a_2 - a_2')\frac{1}{T^2} + (a_3 - a_3')\frac{1}{T^3} . \qquad (21)$$

The effective transmittancy of a smoked glass in the light of two color filters is presented in Figure 4 as a function of the temperature. It is experimentally determined exactly as is that of neutral attenuation media in section (2a).

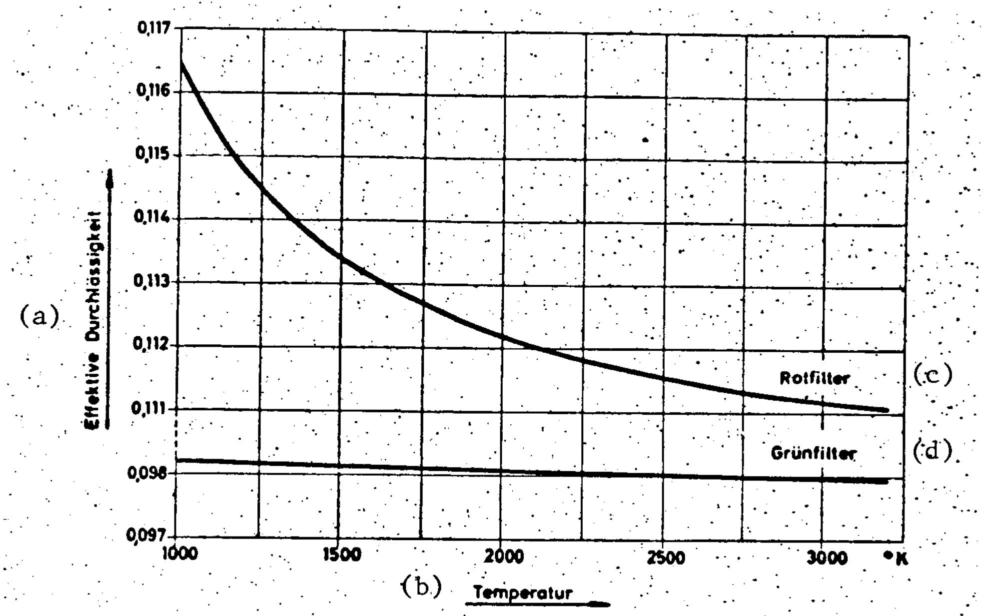

Figure 4

Effective Transmittancy of Smoked Glass in the Light of Two Filters
(a)-Effective Transmittancy; (b)-Temperature;
(c)-Red Filter; (d)-Green Filter

Effective Wavelength

Effective wavelength λ_{eff} is ordinarily defined by the equation

$$\frac{Mc_2}{\lambda_{eff}}\left(\frac{1}{T}-\frac{1}{T'}\right)=\log\frac{\int_\lambda P(\lambda,T')\,V(\lambda)\,D(\lambda)\,d\lambda}{\int_\lambda P(\lambda,T)\,V(\lambda)\,D(\lambda)\,d\lambda}. \qquad (22)$$

Here, the left side of Equation (22) is derived from Wien's approximation [Equation (7)]. As in Equation (20), it is now true that:

$$\frac{Mc_2}{\lambda_{eff}}\left(\frac{1}{T}-\frac{1}{T'}\right)=a_1\left(\frac{1}{T'}-\frac{1}{T}\right)+a_2\left(\frac{1}{T'^2}-\frac{1}{T^2}\right)+a_3\left(\frac{1}{T'^3}-\frac{1}{T^3}\right).$$

For the passage to the limit $T \to T'$, the effective wavelength /650 becomes the effective limiting wavelength λ_w. Application of L'Hospital's rule gives

$$\frac{1}{\lambda_w}=-\frac{a_1}{Mc_2}-2\frac{a_2}{Mc_2}\frac{1}{T}-3\frac{a_3}{Mc_2}\frac{1}{T^2}. \qquad (23)$$

The equation

$$\frac{1}{\lambda_{eff}} = \frac{1}{2}\left[\frac{1}{\lambda_w}(T') + \frac{1}{\lambda_w}(T)\right] + \frac{1}{2}\frac{a_3}{Mc_2}\left(\frac{1}{T} - \frac{1}{T'}\right)^2. \qquad (24)$$

is true of the relationship between the effective limiting wavelength and effective wavelength.

Equation (22) may be regarded as a definition, but a precise formulation would read:

$$\log \frac{e^{\frac{c_2}{T\lambda_{eff}}} - 1}{e^{\frac{c_2}{T'\lambda_{eff}}} - 1} = \log \frac{\int_\lambda P(\lambda, T')\, V(\lambda)\, D(\lambda)\, d\lambda}{\int_\lambda P(\lambda, T)\, V(\lambda)\, D(\lambda)\, d\lambda}. \qquad (25)$$

It follows that:

$$M \ln \frac{e^{\frac{c_2}{T\lambda_{eff}}} - 1}{e^{\frac{c_2}{T'\lambda_{eff}}} - 1} = a_1\left(\frac{1}{T'} - \frac{1}{T}\right) + a_2\left(\frac{1}{T'^2} - \frac{1}{T^2}\right) + a_3\left(\frac{1}{T'^3} - \frac{1}{T^3}\right) \qquad (26)$$

and, finally, for the passage to the limit: /651

$$\frac{1}{\lambda_w} = -\left(\frac{a_1}{Mc_2} + 2\frac{a_2}{Mc_2}\frac{1}{T} + 3\frac{a_3}{Mc_2}\frac{1}{T^2}\right)\left(1 - e^{-\frac{c_2}{\lambda_w T}}\right). \qquad (27)$$

Figure 5 represents λ_w from Equation (23) and Equation (27) for two color filters as a function of temperature.

Conclusion

The computational method described has the essential advantage of achieving high accuracy with a moderate amount of calculation. Only four integrations are required for any combination of filters, nor is it necessary to know the effective wavelength, but it is derived as a subsidiary result. Representation of the logarithm of the luminous-density integrals as a polynomial of the third degree [Equation (9)] leads to a quadratic representation of the effective wavelength [Equation (23)], which is an improvement over the linear form given by Henning (Ref. 1).

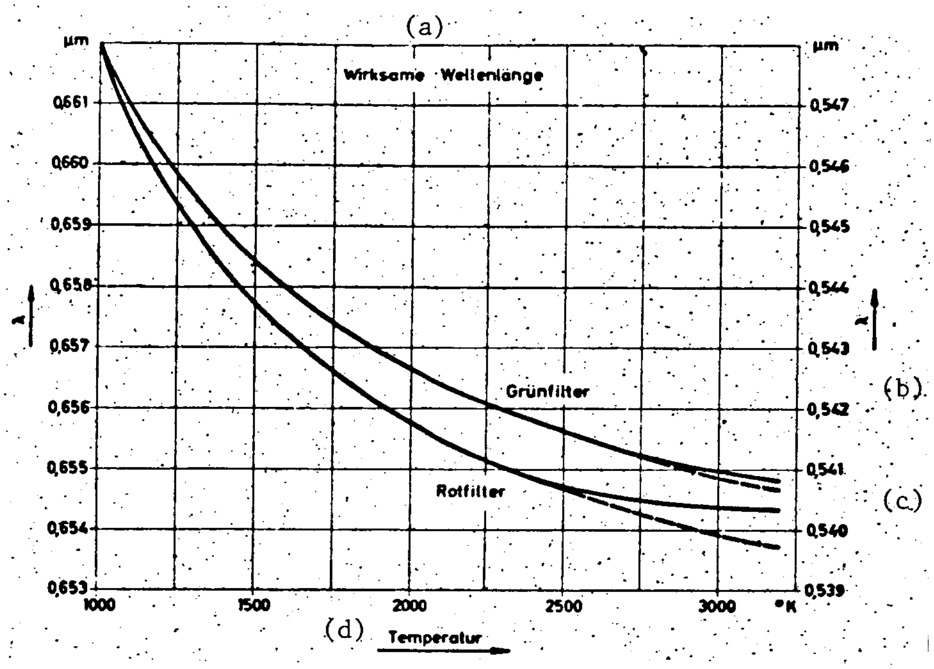

Figure 5

Effective Limiting Wavelength of Two Color Filters

______ Precisely Calculated According to Planck [Equation (27)]

------ Calculated According to Wien [Equation (23)]

(a)-Effective Wavelength; (b)-Green Filter;
(c)-Red Filter; (d)-Temperature

It is a peculiar fact that no extensive conversions are necessitated by establishing a new value for the radiation constant c_2. Instead of expansion with respect to $1/T$ in Equation (8), it may also take place with respect to c_2/T so that:

$$\sum_k \frac{a_k}{T^k} = \sum_k b_k \left(\frac{c_2}{T}\right)^k. \tag{28}$$

holds true.

Comparison of the coefficients gives:

$$a_k = b_k c_2^k, \tag{29}$$

where the values of b_k are now invariant with respect to a change in c_2.

If a new value c_2^* is now introduced, the new constants a_k^* ensue from the old value of a_k from the relationship:

$$a_k^* = a_k \left(\frac{c_2^*}{c_2} \right)^k. \tag{30}$$

Accuracy may be increased at will by continuing the series expansion in Equation (9), or the temperature range may be widened. The polynomial representation is well suited as an interpolation formula for tabulations by means of electronic computers.

The discussion of non-black radiating bodies will follow at a later period.

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